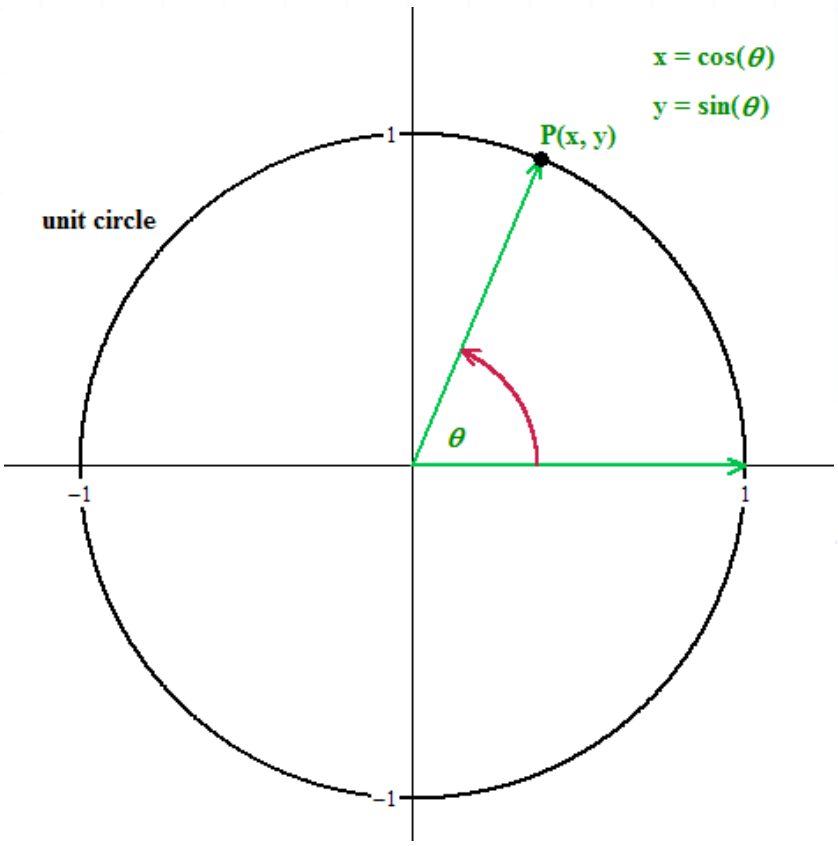
Finding sine and cosine of an angle theta

Imagine an angle θ in standard position where P is a point on its terminal side, and where P lies on the unit circle, which is the circle centered at the origin with radius 1. Then the coordinates of P are (x, y), and

$$x = \cos\theta$$

$$y = \sin\theta$$

Here's a picture of that:



No matter what angle you pick along the circle (remember that the center of the circle is the origin and that the radius of the circle is 1), the x- and y-coordinates of every point that's on the circle will satisfy the equation $x^2 + y^2 = 1$. Also, the value of x is always in the interval $[-1,1]$, and the value of y is also always in the interval $[-1,1]$.

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As an example, at $\theta = 0°$,

$$\cos(0°) = 1$$

$$\sin(0°) = 0$$

which makes sense since the point on the circle at $\theta = 0°$ is on the positive side of the x-axis at $(1,0)$.

To take another example, at $\theta = \pi/2$ radians,

$$\cos\left(\frac{\pi}{2}\right) = 0$$

$$\sin\left(\frac{\pi}{2}\right) = 1$$

which makes sense since the point on the circle at $\theta = \pi/2$ is on the positive side of the y-axis at $(0,1)$.

Example

Find $\sin(45°)$ and $\cos(45°)$.

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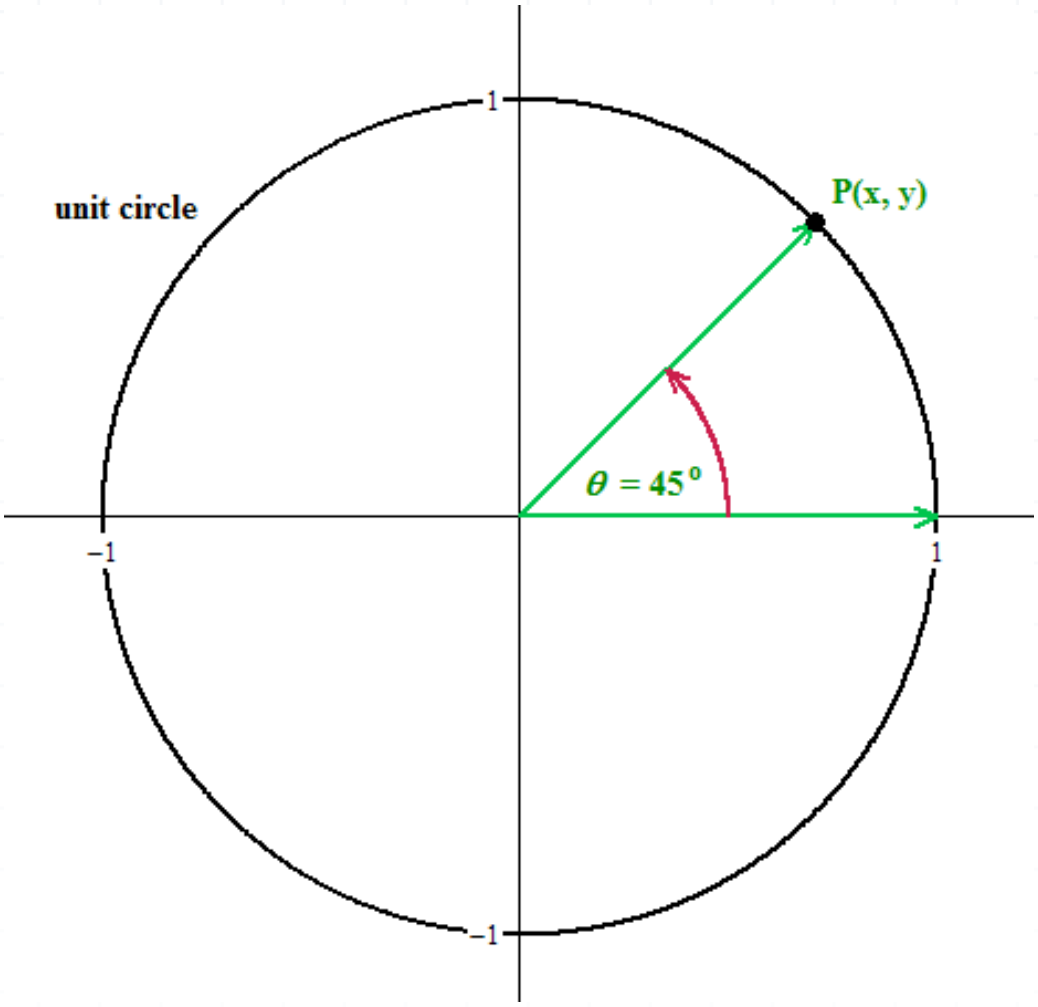

Since $45°$ is exactly halfway between $0°$ and $90°$, we know that the x-coordinate of the point P is equal to the y-coordinate of P. Because

$$x^2 + y^2 = 1$$

and since $x = y$ when $\theta = 45°$, we can substitute x for y and get

$$x^2 + x^2 = 1$$

$$2x^2 = 1$$

$$x^2 = \frac{1}{2}$$

$$x = \pm\sqrt{\frac{1}{2}}$$

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But we know we're in the first quadrant, which means x must be positive, so

$$x = \sqrt{\frac{1}{2}}$$

$$x = \frac{\sqrt{1}}{\sqrt{2}}$$

$$x = \frac{1}{\sqrt{2}}$$

We'll rationalize the denominator.

$$x = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

$$x = \frac{\sqrt{2}}{2}$$

Because x and y have the same value, we also know that

$$y = \frac{\sqrt{2}}{2}$$

Therefore,

$$\sin(45^\circ) = \frac{\sqrt{2}}{2}$$

$$\cos(45^\circ) = \frac{\sqrt{2}}{2}$$

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We can create a table for some of the values of $\sin$ and $\cos$.

θ **in degrees**	θ **in radians**	$\cos\theta$ **or** x	$\sin\theta$ **or** y
$0°$	0	1	0
$90°$	$\pi/2$	0	1
$180°$	π	-1	0
$270°$	$3\pi/2$	0	-1
$360°$	2π	1	0

Example

Use a graphical method to estimate the values of $\sin(127°)$ and $\cos(127°)$ to one decimal place.

We'll sketch the sides of an angle of $127°$. Since the angle is larger than $90°$ but less than $180°$, we know it's in the second quadrant. Because $127°$ is $37°$ past $90°$, it's $37/90$ of the way (which is about $41/100$) from $90°$ to $180°$.

We'll graph the unit circle and then estimate the values of the x- and y-coordinates where P on $127°$ intersects the unit circle.

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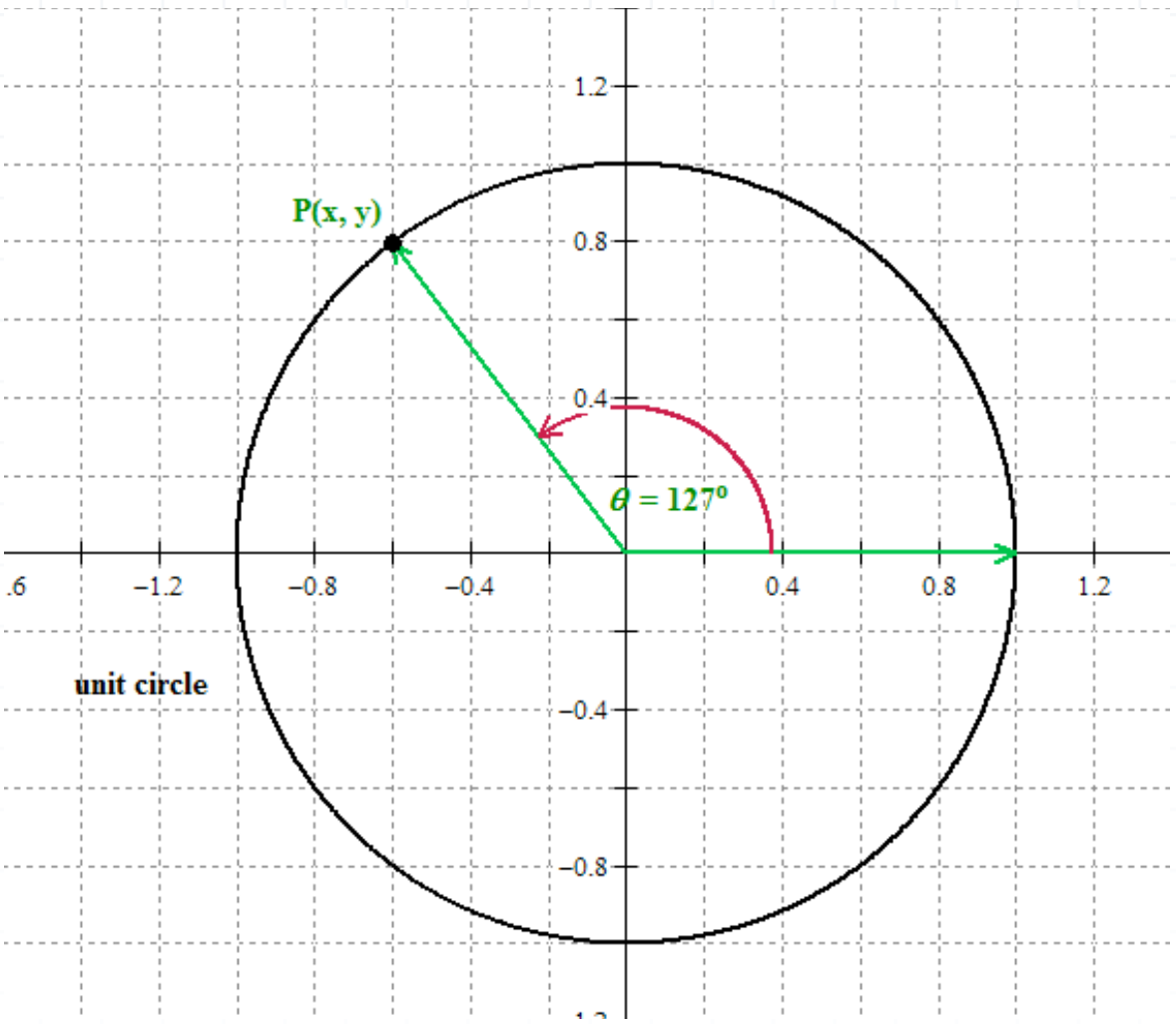

According to the graph, $x \approx -0.6$ and $y \approx 0.8$. Since $\sin(127°) = y$ and $\cos(127°) = x$, we'll say that $\sin(127°) \approx 0.8$ and $\cos(127°) \approx -0.6$.

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